Optical and Quantum Electronics (2023) 55:534
https://doi.org/10.1007/s11082-023-04807-8

Application of the generalized unified method to solve (2+1)-dimensional Kundu–Mukherjee–Naskar equation

Tuğba Aydemir[1]

Received: 20 February 2023 / Accepted: 27 March 2023 / Published online: 11 April 2023

Abstract
The purpose of this study is to introduce a new extension method named the generalized unified method (GUM) and apply this method to the the (2+1) dimensional Kundu–Mukherjee–Naskar (KMN) equation. The GUM as a powerful method provides more general exact solutions for nonlinear partial differential equations (NPDEs) in a compact form with free parameters. Various exact solutions including also hyperbolic, trigonometric and rational forms can be derived from the obtained exact solutions with tuning these free parameters. The reason of choosing the KMN equation is that this equation as extension of the Schrödinger equation is used to model great numbers of considerable physical phenomena such as ion-acoustic waves in oceanic rogue waves, magnetized plasmas, bending of light beams, propagation pulses in optical fiber. Considering the physical importance of the KMN equation, the obtained wide range solution sets by using the GUM will play a significant role in the applied sciences that use the KMN equation to model their problems.

Keywords Kundu–Mukherjee–Naskar equation · The generalized unified method · The unified method · Exact solution

1 Introduction

Nonlinear Partial Differential Equations (NPDEs) have been used for many years in nonlinear science to model various physical phenomena and their process. Over the past few decades, the dynamics of nonlinear waves have appeared in a variety of applications in diverse fields such as transmission in plasma physics, optical communication channels, hydrodynamics, optics, fluid dynamics, quantum mechanics, biology, economics, and also in other branches. Constructing the exact solutions of these equations plays vital role because the problems used to model physical phenomena lead to the discovery of significant inventions which have high positive impact in life. Therefore, various distinct methods are used to solve NPDEs in mathematical studies (Althobaiti et al. 2021; Seadawy et al. 2021; El-Rashidy et al. 2021; Marin et al. 2022; Seadawy and Cheemaa 2020; Wang et al. 2023; Ali et al. 2020). Particularly, the Schrödinger equation is a very common equation which is

---

✉ Tuğba Aydemir
tgb.aydemir@gmail.com

[1] The Faculty of Economics and Administrative Sciences, Yalova University, Yalova, Turkey

describing some problems in nonlinear optics, plasma physics, fluid dynamics, photonics, quantum electronics, water waves, electromagnetism (Sulaiman et al. 2018; Biswas et al. 2017; Bulut et al. 2018; Biswas et al. 2018; Eslami and Neirameh 2018; Nakatsuji et al. 2018; Lan and Guo 2020; Saha et al. 2020; Hollm et al. 2021; Kudryashov 2021; Khalil et al. 2021; Akinyemi et al. 2021; Bilal et al. 2021), and many others. However, in the mathematical field, it is not easy to explain the wave motion deterministically by considering nonlinear interactions in order to understand the behavior of a nonlinear system.

The Kundu–Mukherjee–Naskar (KMN) equation firstly proposed by Anjan Kundu, Abhik Mukherjee and Tapan Naskar in 2013 (Kundu 2013; Kundu et al. 2014) is described an extension of the Schrödinger equation containing mixed types of nonlinear effect in reference to Kerr and non-Kerr law nonlinearities. The structure of the (2+1)-dimensional KMN equation arised as a two-dimensional nonlinear Schrödinger equation derived from the basic hydrodynamic equation is given by the following equation:

$$iu_t + \sigma u_{xy} + i\kappa u(uu_x^* - u^* u_x) = 0, \tag{1}$$

where $u = u(x, y, t)$ is a complex valued function with independent variables x and y spatial variables and t is temporal variable, and asterik(*) indicates complex conjugation. The first term represents the temporal evolution of the pulse, while the second term with the coefficient $\sigma \neq 0$ represents the dispersion term in the KMN equation. The last term with the coefficient $\kappa \neq 0$ is indicates nonlinear effect different from the conventional Kerr law nonlinearity or any known non-Kerr law media. In other words, this term accounts for "current-like" nonlinearity that arises from chirality to explain also the phenomena of bending of light beams.

The KMN equation initially Kundu (2013) is considered to model oceanic rogue waves as well as hole waves in deep sea. Afterwards, many researchers from different fields, particularly physics, are noticed that this equation can be used to describe in many diverse problems such as two-dimensional ion-acoustic waves in magnetized plasmas, (Kundu et al. 2014; Mukherjee and Kundu 2019; Mukherjee et al. 2015; Wen 2017; Haas and Mahmood 2016; Guo et al. 2018), bending of light beams (Ekici et al. 2019), propagation pulses in optical fiber (Yildirim 2019c, a, b).

Due to its physical importance in applied sciences, several different methods have been applied to the KMN equation by many authors to find exact solutions. Some of these methods are the logarithmic transformation method (El-Rashidy and Seadawy 2020), trial equation method (Yildirim 2019c; Biswas et al. 2018), extended trial function method (Ekici et al. 2019), the first integral method (Kudryashov 2019), the method of undetermined coefficients and Lie symmetry (Biswas et al. 2020), the modified simple equation approach (Yildirim 2019a), the new extended algebraic method (Jhangeer et al. 2020), the ansatz approach and the sine Gordon expansion method (Aliyu et al. 2020), the F-expansion and functional variable methods (Yildirim and Mirzazadeh 2020; Rezazadeh et al. 2021), the new extended direct algebraic method (Gunerhan et al. 2020), and the exp-function method (Talarposhti et al. 2020; San et al. 2022), the unified method (Islam et al. 2022). However, considering the importance of this equation in science and engineering, there is still need to have more general solutions with free parameters for the KMN equation (Singh et al. 2023). Therefore, the objective of this study is to propose a new computational method called the generalized unified method(GUM) for constructing more general solutions of the KMN equation. The GUM is an extension of the unified method (Ullah et al. 2021; Bilal et al. 2022; Bilal and Ahmad 2022; Shahen and Rahman 2022; Arafat et al. 2022; Islam et al. 2022, 2022; Uddin et al. 2022; Nandi et al. 2022; Akbulut and Kumar 2022; Akbulut et al. 2023; Kumar et al. 2023; Ullah et al. 2022; Arafat et al. 2022; Bilal and Ahmad 2022b) that provides more general solutions with free parameters. This free parameters allow us not only to have many solutions but also to regulate amplitude and width of the wave solutions to distinguish the physical properties.

This article is structured as follows. Firstly, the algorithm of the generalized unified method (GUM) is described step by step in Sect. 2. Then, the exact solutions of (2+1)-dimensional Kundu–Mukher jee-Naskar equation (KMN) constructed by the GUM are presented in Sect. 3. In Sect. 4, physical structures and graphical illustrations of the some selected solutions for the equation are displayed. Lastly, conclusive remarks are given in Sect. 5.

2 The algorithm of the generalized unified method

The generalized unified method (GUM) is developed from the unified method (Gozukizil et al. 2016; Akcagil and Aydemir 2018) that has been effectively applied by many authors (Ullah et al. 2021; Bilal et al. 2022; Bilal and Ahmad 2022; Shahen and Rahman 2022; Arafat et al. 2022; Islam et al. 2022, 2022; Uddin et al. 2022; Nandi et al. 2022; Akbulut and Kumar 2022; Akbulut et al. 2023; Kumar et al. 2023; Ullah et al. 2022; Arafat et al. 2022; Bilal and Ahmad 2022b) in solving different types of nonlinear partial differential equations (NPDEs) encountered in science and engineering studies. In this section, how to apply the GUM to solve NPDEs is explained step by step. Additionally, the GUM as en effective and simple method can be applied not only NPDEs but also fractional NPDEs with more than three independent variables (Malik et al. 2023; Asjad et al. 2023; Asghari et al. 2023b, a).

Let P represents a general form NPDEs for an unknown function $w = w(x, y, t)$ with three independent variables x, y and t such that highest order derivative and nonlinear terms of w are involved.

$$P(w, w_t, w_x, w_y, w_{xy}, w_{xt}, w_{tt}, w_{xx}, w_{yy}, ...) = 0, \tag{2}$$

where the subscript represents the partial derivative of w with respect to independent variables.

Step 1: In order to reduce NPDE to ordinary differential equation (ODE), substituting $w(x, y, t) = W(\eta)$ into Eq.(2) such that $\eta = x + ry - ct + \eta_0$ is the wave variable where c is the wave velocity determined later and η_0 real arbitrary free parameter. That gives Eq. (2) in the ODE form

$$P(W, W', W'', W''', ...) = 0, \tag{3}$$

where the superscript indicates the derivative of the function W with respect to η.

Step 2: Assume the solution of Eq.(3) can be expressed by an ansatz as follows:

$$W(\eta) = a_0 + \sum_{m=1}^{M} \left[a_m \phi^m + b_m \phi^{-m}\right], \tag{4}$$

where a_m, b_m are the coefficients of ϕ which are determined later. Moreover, $\phi = \phi(\eta)$ satisfies the Riccati differential equation defined below with $\phi' = \frac{d\phi}{d\eta}$ and $\mu = (c + id)$ where c and d are parameters.

$$\phi'(\eta) = \phi^2(\eta) - \mu^2. \tag{5}$$

The general solutions of Eq.(5) as follows:

$$\phi(\eta)=\begin{cases}\phi_1=\dfrac{(c+id)\sqrt{A^2+(B+iC)^2}-A(c+id)\cosh\left(2(c+id)(\eta+\eta_0)\right)}{(B+iC)+A\sinh\left(2(c+id)(\eta+\eta_0)\right)},\\ \phi_2=\dfrac{-(c+id)\sqrt{A^2+(B+iC)^2}-A(c+id)\cosh\left(2(c+id)(\eta+\eta_0)\right)}{(B+iC)+A\sinh\left(2(c+id)(\eta+\eta_0)\right)},\\ \phi_3=\dfrac{(c+id)\left(-A+e^{-2(c+id)(\eta+\eta_0)}\right)}{\left(A+e^{-2(c+id)(\eta+\eta_0)}\right)},\\ \phi_4=\dfrac{-(c+id)\left(-A+e^{2(c+id)(\eta+\eta_0)}\right)}{\left(A+e^{2(c+id)(\eta+\eta_0)}\right)},\\ \phi_5=-\dfrac{1}{\eta+\eta_0}\end{cases}\tag{6}$$

where $A \neq 0$, B and C are real arbitrary parameters.

Step 3: The balance value M in Eq.(4) can be determined by considering the homogeneous balance between the linear term of the highest order with the nonlinear term of highest degree.

Step 4: To obtain a system of algebraic equations that gives the travelling wave solutions of Eq.(2), it is substituted Eq.(4) and its derivatives into Eq.(3). As calculating the derivatives of W, considering Eq. (5) facilitates solving process on computer. It yields a polynomial in powers ϕ.

Step 5: After solving the obtained a system of algebraic equations to find the coefficients of Eq.(4), these values are substituted into Eq.(4), then the travelling wave solutions of Eq.(2) are obtained in closed form with free parameters A, B and C.

3 Solutions of the Kundu–Mukherjee–Naskar equation

This section contains general solutions of the Kundu–Mukherjee -Naskar Equation (KMN) which has been obtained by applying the generalized unified method (GUM). The KMN equation is given by

$$iu_t+\sigma u_{xy}+i\kappa u(uu_x^*-u^*u_x)=0.\tag{7}$$

Firstly, in order to reduce the KMN equation to an nonlinear ordinary differential equation form, the wave transformation $u(x,y,t)=U(\eta)e^{-i\Omega}$ is applied. In this transformation, $U(\eta)$ denotes the shape of the nonlinear wave with the wave variable $\eta=x+ry-ct+\eta_0$, and the phase of wave $\Omega=px+qy-vt+\Omega_0$, where Ω_0, η_0 are arbitrary free parameters. Substituting $u(x,y,t)=U(\eta)e^{-i\Omega}$, $u^*(x,y,t)=U(\eta)e^{i\Omega}$ and its derivatives

$$\begin{aligned}&u_t(x,y,t)=e^{-i\Omega}(ivU(\eta)-cU'(\eta))\\&u_x(x,y,t)=e^{-i\Omega}(-ipU(\eta)+U'(\eta))\\&u_x^*(x,y,t)=e^{i\Omega}(ipU(\eta)+U'(\eta))\\&u_{xy}(x,y,t)=e^{-i\Omega}(-pqU(\eta)-i(pr+q)U'(\eta)+rU''(\eta))\end{aligned}$$

into Eq.(7), then the obtained nonlinear ordinary differential equation as belows:

$$-(pq\sigma+v)U+r\sigma U''-2p\kappa U^3-i(c+(pr+q)\sigma)U'=0.\tag{8}$$

Decomposing to real and imaginary parts of Eq.(8) is resulted in $c=-(pr+q)\sigma$ from imaginary parts, and the following reduced nonlinear ordinary differential equation for the KMN equation from real parts.

$$2p\kappa U^3 - r\sigma U'' + (pq\sigma + v)U' = 0. \tag{9}$$

To simplify the computation at the computer, the coefficients of Eq.(9) are defined $D = 2p\kappa$, $E = r\sigma$ and $F = (pq\sigma + v)$. Hence, Eq.(9) is stated as follows:

$$DU^3 - EU'' + FU' = 0. \tag{10}$$

Equating a balance between the highest order U'' with the nonlinear term U^3 gives the simple equation $M + 2 = 3M$. So the solutions of the KMN equation in (7) can be written in the form with this balance value $M = 1$.

$$U(\eta) = a_0 + a_1\phi + \frac{b_1}{\phi}, \tag{11}$$

where a_0, a_1 and b_1 are coefficients of ϕ which are determined later. Substituting Eq.(11) and its derivatives into Eq.(10), then equating the coefficients of different power of ϕ to zero gives a system of nonlinear algebraic equations with a_0, a_1, b_1 and μ. Solving this algebraic equations system by using Maple yields the following sets of parameters:

Set 1.

$a_0 = 0, \quad a_1 = \sqrt{\frac{2E}{D}} \quad b_1 = 0, \quad \mu = \sqrt{\frac{-F}{2E}},$

Set 2.

$a_0 = 0, \quad a_1 = -\sqrt{\frac{2E}{D}} \quad b_1 = 0, \quad \mu = \sqrt{\frac{-F}{2E}},$

Set 3.

$a_0 = 0, \quad a_1 = \sqrt{\frac{2E}{D}} \quad b_1 = 0, \quad \mu = -\sqrt{\frac{-F}{2E}},$

Set 4.

$a_0 = 0, \quad a_1 = -\sqrt{\frac{2E}{D}} \quad b_1 = 0, \quad \mu = -\sqrt{\frac{-F}{2E}},$

Set 5.

$a_0 = 0, \quad a_1 = 0, \quad b_1 = \frac{F}{\sqrt{2DE}} \quad \mu = \sqrt{\frac{-F}{2E}},$

Set 6.

$a_0 = 0, \quad a_1 = 0, \quad b_1 = -\frac{F}{\sqrt{2DE}} \quad \mu = \sqrt{\frac{-F}{2E}},$

Set 7.

$a_0 = 0, \quad a_1 = 0, \quad b_1 = \frac{F}{\sqrt{2DE}} \quad \mu = -\sqrt{\frac{-F}{2E}},$

Set 8.

$a_0 = 0, \quad a_1 = 0, \quad b_1 = -\frac{F}{\sqrt{2DE}} \quad \mu = -\sqrt{\frac{-F}{2E}},$

Set 9.

$a_0 = 0, \quad a_1 = -\sqrt{\frac{2E}{D}}, \quad b_1 = \frac{F}{\sqrt{8DE}} \quad \mu = \frac{1}{2}\sqrt{\frac{F}{E}},$

Set 10.

$a_0 = 0, \quad a_1 = \sqrt{\frac{2E}{D}}, \quad b_1 = -\frac{F}{\sqrt{8DE}} \quad \mu = \frac{1}{2}\sqrt{\frac{F}{E}},$

Set 11.

$a_0 = 0, \quad a_1 = -\sqrt{\frac{2E}{D}}, \quad b_1 = \frac{F}{\sqrt{8DE}} \quad \mu = -\frac{1}{2}\sqrt{\frac{F}{E}},$

Set 12.

$a_0 = 0, \quad a_1 = \sqrt{\frac{2E}{D}}, \quad b_1 = -\frac{F}{\sqrt{8DE}} \quad \mu = -\frac{1}{2}\sqrt{\frac{F}{E}},$

Set 13.

$a_0 = 0, \quad a_1 = -\sqrt{\frac{E}{2D}}, \quad b_1 = \frac{F}{\sqrt{32DE}} \quad \mu = \sqrt{\frac{-F}{8E}},$

Set 14.

$a_0 = 0, \quad a_1 = \sqrt{\frac{E}{2D}}, \quad b_1 = -\frac{F}{\sqrt{32DE}} \quad \mu = \sqrt{\frac{-F}{8E}},$

Set 15.

$a_0 = 0, \quad a_1 = -\sqrt{\frac{E}{2D}}, \quad b_1 = \frac{F}{\sqrt{32DE}} \quad \mu = -\sqrt{\frac{-F}{8E}},$

Set 16.

$a_0 = 0, \quad a_1 = \sqrt{\frac{E}{2D}}, \quad b_1 = -\frac{F}{\sqrt{32DE}} \quad \mu = -\sqrt{\frac{-F}{8E}}.$

After applying this method to the KMN equation, the following solutions are derived from (11) using these 16 coefficient sets along with Eq.(6). Throughout this section, the first index n indicates which one of ϕ solution in (6) is used and the second index m shows the set number above is used in $u_{n,m}$. In this context, substituting the coefficients from set 1 to set 16 with ϕ_1 into (11) gives solutions are respectively as follows:

$$u_{1,\{1,2\}}(x,y,t) = \frac{\mp\sqrt{\frac{-F}{D}}\left(\sqrt{A^2+(B+iC)^2} - A\cosh\left(\sqrt{\frac{-2F}{E}}(\eta+\eta_0)\right)\right)}{(B+iC) + A\sinh\left(\sqrt{\frac{-2F}{E}}(\eta+\eta_0)\right)} e^{-i\Omega}, \quad (12)$$

$$u_{1,\{3,4\}}(x,y,t) = \frac{\pm\sqrt{\frac{-F}{D}}\left(\sqrt{A^2+(B+iC)^2} - A\cosh\left(\sqrt{\frac{-2F}{E}}(\eta+\eta_0)\right)\right)}{(B+iC) - A\sinh\left(\sqrt{\frac{-2F}{E}}(\eta+\eta_0)\right)} e^{-i\Omega}, \quad (13)$$

$$u_{1,\{5,6\}}(x,y,t) = \frac{\mp\sqrt{\frac{-F}{D}}\left((B+iC) + A\sinh\left(\sqrt{\frac{-2F}{E}}(\eta+\eta_0)\right)\right)}{\sqrt{A^2+(B+iC)^2} - A\cosh\left(\sqrt{\frac{-2F}{E}}(\eta+\eta_0)\right)} e^{-i\Omega}, \quad (14)$$

$$u_{1,\{7,8\}}(x,y,t) = \frac{\pm\sqrt{\frac{-F}{D}}\left((B+iC) - A\sinh\left(\sqrt{\frac{-2F}{E}}(\eta+\eta_0)\right)\right)}{\sqrt{A^2+(B+iC)^2} - A\cosh\left(\sqrt{\frac{-2F}{E}}(\eta+\eta_0)\right)} e^{-i\Omega}, \quad (15)$$

$$u_{1,\{9,10\}}(x,y,t) = \mp\sqrt{\frac{F}{2D}}\left(\frac{\left((B+iC)+A\sinh\left(\sqrt{\frac{F}{E}}(\eta+\eta_0)\right)\right)}{\sqrt{A^2+(B+iC)^2}-A\cosh\left(\sqrt{\frac{F}{E}}(\eta+\eta_0)\right)} - \frac{\left(\sqrt{A^2+(B+iC)^2}-A\cosh\left(\sqrt{\frac{F}{E}}(\eta+\eta_0)\right)\right)}{(B+iC)+A\sinh\left(\sqrt{\frac{F}{E}}(\eta+\eta_0)\right)}\right) e^{-i\Omega}, \quad (16)$$

$$u_{1,\{11,12\}}(x,y,t) = \pm\sqrt{\frac{F}{2D}}\left(-\frac{\left((B+iC)-A\sinh\left(\sqrt{\frac{F}{E}}(\eta+\eta_0)\right)\right)}{\sqrt{A^2+(B+iC)^2}-A\cosh\left(\sqrt{\frac{F}{E}}(\eta+\eta_0)\right)} + \frac{\left(\sqrt{A^2+(B+iC)^2}-A\cosh\left(\sqrt{\frac{F}{E}}(\eta+\eta_0)\right)\right)}{(B+iC)-A\sinh\left(\sqrt{\frac{F}{E}}(\eta+\eta_0)\right)}\right) e^{-i\Omega}, \quad (17)$$

$$u_{1,\{13,14\}}(x,y,t) = \mp\frac{1}{2}\sqrt{\frac{F}{D}}\left(\frac{\left((B+iC)+A\sinh\left(\sqrt{-\frac{F}{2E}}(\eta+\eta_0)\right)\right)}{\sqrt{A^2+(B+iC)^2}-A\cosh\left(\sqrt{-\frac{F}{2E}}(\eta+\eta_0)\right)} - \frac{\left(\sqrt{A^2+(B+iC)^2}-A\cosh\left(\sqrt{-\frac{F}{2E}}(\eta+\eta_0)\right)\right)}{(B+iC)+A\sinh\left(\sqrt{-\frac{F}{2E}}(\eta+\eta_0)\right)}\right)e^{-i\Omega}, \tag{18}$$

$$u_{1,\{15,16\}}(x,y,t) = \pm\frac{1}{2}\sqrt{\frac{F}{D}}\left(\frac{\left((B+iC)-A\sinh\left(\sqrt{-\frac{F}{2E}}(\eta+\eta_0)\right)\right)}{\sqrt{A^2+(B+iC)^2}-A\cosh\left(\sqrt{-\frac{F}{2E}}(\eta+\eta_0)\right)} - \frac{\left(\sqrt{A^2+(B+iC)^2}-A\cosh\left(\sqrt{-\frac{F}{2E}}(\eta+\eta_0)\right)\right)}{(B+iC)-A\sinh\left(\sqrt{-\frac{F}{2E}}(\eta+\eta_0)\right)}\right)e^{-i\Omega}. \tag{19}$$

Substituting the coefficients from set 1 to set 16 with ϕ_2 into (11) gives the following solutions:

$$u_{2,\{1,2\}}(x,y,t) = \frac{\pm\sqrt{\frac{-F}{D}}\left(\sqrt{A^2+(B+iC)^2}+A\cosh\left(\sqrt{\frac{-2F}{E}}(\eta+\eta_0)\right)\right)}{(B+iC)+A\sinh\left(\sqrt{\frac{-2F}{E}}(\eta+\eta_0)\right)}e^{-i\Omega}, \tag{20}$$

$$u_{2,\{3,4\}}(x,y,t) = \frac{\mp\sqrt{\frac{-F}{D}}\left(\sqrt{A^2+(B+iC)^2}+A\cosh\left(\sqrt{\frac{-2F}{E}}(\eta+\eta_0)\right)\right)}{(B+iC)-A\sinh\left(\sqrt{\frac{-2F}{E}}(\eta+\eta_0)\right)}e^{-i\Omega}, \tag{21}$$

$$u_{2,\{5,6\}}(x,y,t) = \frac{\pm\sqrt{\frac{-F}{D}}\left((B+iC)+A\sinh\left(\sqrt{\frac{-2F}{E}}(\eta+\eta_0)\right)\right)}{\sqrt{A^2+(B+iC)^2}+A\cosh\left(\sqrt{\frac{-2F}{E}}(\eta+\eta_0)\right)}e^{-i\Omega}, \tag{22}$$

$$u_{2,\{7,8\}}(x,y,t) = \frac{\mp\sqrt{\frac{-F}{D}}\left((B+iC)-A\sinh\left(\sqrt{\frac{-2F}{E}}(\eta+\eta_0)\right)\right)}{\sqrt{A^2+(B+iC)^2}+A\cosh\left(\sqrt{\frac{-2F}{E}}(\eta+\eta_0)\right)}e^{-i\Omega}, \tag{23}$$

$$u_{2,\{9,10\}}(x,y,t) = \pm\sqrt{\frac{F}{2D}}\left(\frac{\left((B+iC)+A\sinh\left(\sqrt{\frac{F}{E}}(\eta+\eta_0)\right)\right)}{\sqrt{A^2+(B+iC)^2}+A\cosh\left(\sqrt{\frac{F}{E}}(\eta+\eta_0)\right)} - \frac{\left(\sqrt{A^2+(B+iC)^2}+A\cosh\left(\sqrt{\frac{F}{E}}(\eta+\eta_0)\right)\right)}{(B+iC)+A\sinh\left(\sqrt{\frac{F}{E}}(\eta+\eta_0)\right)}\right)e^{-i\Omega}, \tag{24}$$

$$u_{2,\{11,12\}}(x,y,t) = \mp\sqrt{\frac{F}{2D}}\left(\frac{\left((B+iC)-A\sinh\left(\sqrt{\frac{F}{E}}(\eta+\eta_0)\right)\right)}{\sqrt{A^2+(B+iC)^2}+A\cosh\left(\sqrt{\frac{F}{E}}(\eta+\eta_0)\right)} - \frac{\left(\sqrt{A^2+(B+iC)^2}+A\cosh\left(\sqrt{\frac{F}{E}}(\eta+\eta_0)\right)\right)}{(B+iC)-A\sinh\left(\sqrt{\frac{F}{E}}(\eta+\eta_0)\right)}\right)e^{-i\Omega}, \tag{25}$$

$$u_{2,\{13,14\}}(x,y,t) = \pm\frac{1}{2}\sqrt{\frac{F}{D}}\left(\frac{\left((B+iC)+A\sinh\left(\sqrt{-\frac{F}{2E}}(\eta+\eta_0)\right)\right)}{\sqrt{A^2+(B+iC)^2}+A\cosh\left(\sqrt{-\frac{F}{2E}}(\eta+\eta_0)\right)} - \frac{\left(\sqrt{A^2+(B+iC)^2}+A\cosh\left(\sqrt{-\frac{F}{2E}}(\eta+\eta_0)\right)\right)}{(B+iC)+A\sinh\left(\sqrt{-\frac{F}{2E}}(\eta+\eta_0)\right)}\right)e^{-i\Omega}, \tag{26}$$

$$u_{2,\{15,16\}}(x,y,t)=\mp\frac{1}{2}\sqrt{\frac{F}{D}}\left(\frac{\left((B+iC)-A\sinh\left(\sqrt{-\frac{F}{2E}}(\eta+\eta_0)\right)\right)}{\sqrt{A^2+(B+iC)^2}+A\cosh\left(\sqrt{-\frac{F}{2E}}(\eta+\eta_0)\right)}-\frac{\left(\sqrt{A^2+(B+iC)^2}+A\cosh\left(\sqrt{-\frac{F}{2E}}(\eta+\eta_0)\right)\right)}{(B+iC)-A\sinh\left(\sqrt{-\frac{F}{2E}}(\eta+\eta_0)\right)}\right)e^{-i\Omega}. \tag{27}$$

Substituting the coefficients from set 1 to set 16 with ϕ_3 and ϕ_4 into (11), that gives

$$u_{3,\{1,2\}}(x,y,t)=\frac{\mp\sqrt{\frac{-F}{D}}\left(-A+e^{-\sqrt{\frac{-2F}{E}}(\eta+\eta_0)}\right)}{\left(A+e^{-\sqrt{\frac{-2F}{E}}(\eta+\eta_0)}\right)}e^{-i\Omega}, \tag{28}$$

$$u_{3,\{3,4\}}(x,y,t)=\frac{\pm\sqrt{\frac{-F}{D}}\left(-A+e^{\sqrt{\frac{-2F}{E}}(\eta+\eta_0)}\right)}{\left(A+e^{\sqrt{\frac{-2F}{E}}(\eta+\eta_0)}\right)}e^{-i\Omega}, \tag{29}$$

$$u_{3,\{5,6\}}(x,y,t)=\frac{\mp\sqrt{\frac{-F}{D}}\left(A+e^{-\sqrt{\frac{-2F}{E}}(\eta+\eta_0)}\right)}{\left(-A+e^{-\sqrt{\frac{-2F}{E}}(\eta+\eta_0)}\right)}e^{-i\Omega}, \tag{30}$$

$$u_{3,\{7,8\}}(x,y,t)=\frac{\pm\sqrt{\frac{-F}{D}}\left(A+e^{\sqrt{\frac{-2F}{E}}(\eta+\eta_0)}\right)}{\left(-A+e^{\sqrt{\frac{-2F}{E}}(\eta+\eta_0)}\right)}e^{-i\Omega}, \tag{31}$$

$$u_{3,\{9,10\}}(x,y,t)=\mp\sqrt{\frac{F}{2D}}\left(\frac{A+e^{-\sqrt{\frac{F}{E}}(\eta+\eta_0)}}{-A+e^{-\sqrt{\frac{F}{E}}(\eta+\eta_0)}}-\frac{-A+e^{-\sqrt{\frac{F}{E}}(\eta+\eta_0)}}{A+e^{-\sqrt{\frac{F}{E}}(\eta+\eta_0)}})\right)e^{-i\Omega}, \tag{32}$$

$$u_{3,\{11,12\}}(x,y,t)=\pm\sqrt{\frac{F}{2D}}\left(-\frac{A+e^{\sqrt{\frac{F}{E}}(\eta+\eta_0)}}{-A+e^{\sqrt{\frac{F}{E}}(\eta+\eta_0)}}+\frac{-A+e^{\sqrt{\frac{F}{E}}(\eta+\eta_0)}}{A+e^{\sqrt{\frac{F}{E}}(\eta+\eta_0)}})\right)e^{-i\Omega}, \tag{33}$$

$$u_{3,\{13,14\}}(x,y,t)=\mp\frac{1}{2}\sqrt{\frac{F}{D}}\left(\frac{A+e^{-\sqrt{\frac{-F}{2E}}(\eta+\eta_0)}}{-A+e^{-\sqrt{\frac{-F}{2E}}(\eta+\eta_0)}}-\frac{-A+e^{-\sqrt{\frac{-F}{2E}}(\eta+\eta_0)}}{A+e^{-\sqrt{\frac{-F}{2E}}(\eta+\eta_0)}})\right)e^{-i\Omega}, \tag{34}$$

$$u_{3,\{15,16\}}(x,y,t)=\pm\frac{1}{2}\sqrt{\frac{F}{D}}\left(-\frac{A+e^{\sqrt{\frac{-F}{2E}}(\eta+\eta_0)}}{-A+e^{\sqrt{\frac{-F}{2E}}(\eta+\eta_0)}}+\frac{-A+e^{\sqrt{\frac{-F}{2E}}(\eta+\eta_0)}}{A+e^{\sqrt{\frac{-F}{2E}}(\eta+\eta_0)}})\right)e^{-i\Omega}. \tag{35}$$

Substituting the coefficients from set 1 to set 4 with ϕ_5 into (11), that gives

$$u_{5,\{1,2\}}(x,y,t) = \pm \frac{\sqrt{\frac{2E}{D}}}{\eta + \eta_0} e^{-i\Omega}. \tag{36}$$

In last part, because of the remaining solutions are trivial solutions, it is obtained only two solutions under the constraint $pq\sigma + v = 0$.

4 Result and discussion

In the previous section, 50 different solutions for the Kundu–Mukherjee–Naskar Equation (KMN) are obtained by applying the generalized unified method (GUM). Firstly, all solutions are summarized as the following manner. It is obvious that by changing the sign of the following solutions, 40 solutions from u_1, u_2, u_3, u_4,u_5, 8 solutions from u_6,u_7, and 2 solutions from u_8 can be derived as in the third section.

$$u_1(x,y,t) = \frac{\pm\sqrt{\frac{-F}{D}}\left(\sqrt{A^2 + (B+iC)^2} \pm A\cosh\left(\sqrt{\frac{-2F}{E}}(\eta+\eta_0)\right)\right)}{(B+iC) \pm A\sinh\left(\sqrt{\frac{-2F}{E}}(\eta+\eta_0)\right)} e^{-i\Omega}, \tag{37}$$

$$u_2(x,y,t) = \frac{\pm\sqrt{\frac{-F}{D}}\left((B+iC) \pm A\sinh\left(\sqrt{\frac{-2F}{E}}(\eta+\eta_0)\right)\right)}{\sqrt{A^2 + (B+iC)^2} \pm A\cosh\left(\sqrt{\frac{-2F}{E}}(\eta+\eta_0)\right)} e^{-i\Omega}, \tag{38}$$

$$u_3(x,y,t) = \pm\sqrt{\frac{F}{2D}}\left[\frac{\left((B+iC) \pm A\sinh\left(\sqrt{\frac{F}{E}}(\eta+\eta_0)\right)\right)}{\sqrt{A^2+(B+iC)^2} \pm A\cosh\left(\sqrt{\frac{F}{E}}(\eta+\eta_0)\right)} - \left(\frac{\left((B+iC) \pm A\sinh\left(\sqrt{\frac{F}{E}}(\eta+\eta_0)\right)\right)}{\sqrt{A^2+(B+iC)^2} \pm A\cosh\left(\sqrt{\frac{F}{E}}(\eta+\eta_0)\right)}\right)^{-1}\right] e^{-i\Omega}, \tag{39}$$

$$u_4(x,y,t) = \pm\frac{1}{2}\sqrt{\frac{F}{D}}\left[\frac{\left((B+iC) \pm A\sinh\left(\sqrt{-\frac{F}{2E}}(\eta+\eta_0)\right)\right)}{\sqrt{A^2+(B+iC)^2} \pm A\cosh\left(\sqrt{-\frac{F}{2E}}(\eta+\eta_0)\right)} - \left(\frac{\left((B+iC) \pm A\sinh\left(\sqrt{-\frac{F}{2E}}(\eta+\eta_0)\right)\right)}{\sqrt{A^2+(B+iC)^2} \pm A\cosh\left(\sqrt{-\frac{F}{2E}}(\eta+\eta_0)\right)}\right)^{-1}\right] e^{-i\Omega}, \tag{40}$$

$$u_5(x,y,t) = \pm\frac{\sqrt{\frac{-F}{D}}\left(\pm A + e^{\pm\sqrt{\frac{-2F}{E}}(\eta+\eta_0)}\right)}{\left(\mp A + e^{\pm\sqrt{\frac{-2F}{E}}(\eta+\eta_0)}\right)} e^{-i\Omega}, \tag{41}$$

$$u_6(x,y,t) = \pm\sqrt{\frac{F}{2D}}\left[\frac{A+e^{\pm\sqrt{\frac{F}{E}}(\eta+\eta_0)}}{-A+e^{\pm\sqrt{\frac{F}{E}}(\eta+\eta_0)}} - \left(\frac{A+e^{\pm\sqrt{\frac{F}{E}}(\eta+\eta_0)}}{-A+e^{\pm\sqrt{\frac{F}{E}}(\eta+\eta_0)}}\right)^{-1}\right] e^{-i\Omega}, \tag{42}$$

$$u_7(x,y,t) = \pm\frac{1}{2}\sqrt{\frac{F}{D}}\left[\frac{A+e^{\pm\sqrt{\frac{-F}{2E}}(\eta+\eta_0)}}{-A+e^{\pm\sqrt{\frac{-F}{2E}}(\eta+\eta_0)}} - \left(\frac{A+e^{\pm\sqrt{\frac{-F}{2E}}(\eta+\eta_0)}}{-A+e^{\pm\sqrt{\frac{-F}{2E}}(\eta+\eta_0)}}\right) - 1\right] e^{-i\Omega}, \tag{43}$$

$$u_8(x,y,t) = \pm\frac{\sqrt{\frac{2E}{D}}}{\eta+\eta_0} e^{-i\Omega}. \tag{44}$$

Several different techniques for solving the NPDEs have been mentioned in the first section, which mostly give solutions in hyperbolic, trigonometric and rational forms. It should be noted here that the solutions obtained by applying GUM already contain these solution forms. While setting $B = 0, C = 0$ and substituting the hyperbolic identities $\cosh(2a) = \cosh^2(a) + \sinh^2(a) = 2\cosh^2(a) - 1 = 2\sinh^2(a) + 1$ and $\sinh(2a) = 2\sinh(a)\cosh(a)$ into the solutions u_1 and u_2 gives the solutions in tanh and coth forms, respectively. The similar manner, setting $A = 1$ in u_5 also gives the solutions in tanh and coth forms, too. Therefore, the GUM with free parameters provides more general solution forms including tanh and coth forms than other techniques. On the other hand, the solutions in sech and cosech forms are also obtained easily by considering the hyperbolic equality $\coth(a) - \tanh(a) = 2\text{cosech}(2\text{a})$ in u_3 and u_4 while setting $B = 0, C = 0$. The similar way, setting $A = 1$ in u_6 and u_7 also gives the solutions in sech and cosech forms. Moreover, considering these hyperbolic-trigonometric equalities $\sinh(ix) = i\sin(x)$, $\cosh(ix) = \cos(x)$, $\sinh(x) = i\sin(ix)$, $\cosh(x) = \cos(ix)$, the obtained solutions are converted easily from hyperbolic to trigonometric or vice versa. It is concluded that the GUM produces many more solutions in compact form with free parameters.

The 3-D graphs of the obtained solutions u_1, u_3, u_5, u_8 for the real part are shown in Fig. 1. The graphs are plotted in the x direction for the parameters $p = 1,\ q = 2,\ r = 3,\ \sigma = 1,\ \kappa = 1,\ v = 1,\ \ A = 10,\ \ B = 1,\ \ C = 1$ over the interval $-10 < x < 10, -10 < t < 10$. The graphs show how diverse characteristics of the solutions obtained by GUM are such as rogue waves waves and periodic waves. In particular, rogue waves called also killer or monster waves have been arised liquid helium, in nonlinear optics, and in microwave cavities other than water (Helal 2022). These type of waves are very significant to model wave phenomenon in applied sciences which are using wave such as optic and ocean engineering.

The 3-D graphs of the obtained solutions u_2, u_4, u_6, u_7 for the real part are shown in Fig. 2. The graphs are plotted in the x direction for the parameters $p = 1,\ q = 2,\ r = 3,\ \sigma = 1,\ \kappa = 1,\ v = 1,\ \ A = 10,\ \ B = 0,\ \ C = 0$ over the interval $-10 < x < 10,\ -10 < t < 10$ in Fig. 2. The graphs show different characteristics of the solutions obtained by GUM. Particularly, the special rogue waves with multiple higher peaks can be forecasted and tracked where and when they emerge using these main solutions with different free parameters.

The 3-D graphs of the obtained solutions u_1, u_3, u_5, u_8 for the real part are shown in Fig. 3. The graphs are plotted in the x direction for the parameters $p = 1,\ q = 2,\ r = 3,\ \sigma = 1,\ \kappa = 1,\ v = 1,\ \ A = 10,\ \ B = 0,\ \ C = 0$ over the interval $-10 < x < 10, -10 < t < 10$ in Fig. 3. Particularly, the GUM provides a unified method to have solutions the rogue type multiple lump wave solutions such as in Fig. 3.

The 3-D graphs of the obtained solutions u_2, u_4, u_6, u_7 for the real part are shown in Fig. 4. The graphs are plotted in the x direction for the parameters $p = 1,\ q = 2,\ r = 3,\ \sigma = 1,\ \kappa = 1,\ v = 1,\ \ A = 10,\ \ B = 0,\ \ C = 0$ over the interval $-10 < x < 10,\ -10 < t < 10$ in Fig. 4. These graphs demonstrate more stable and more

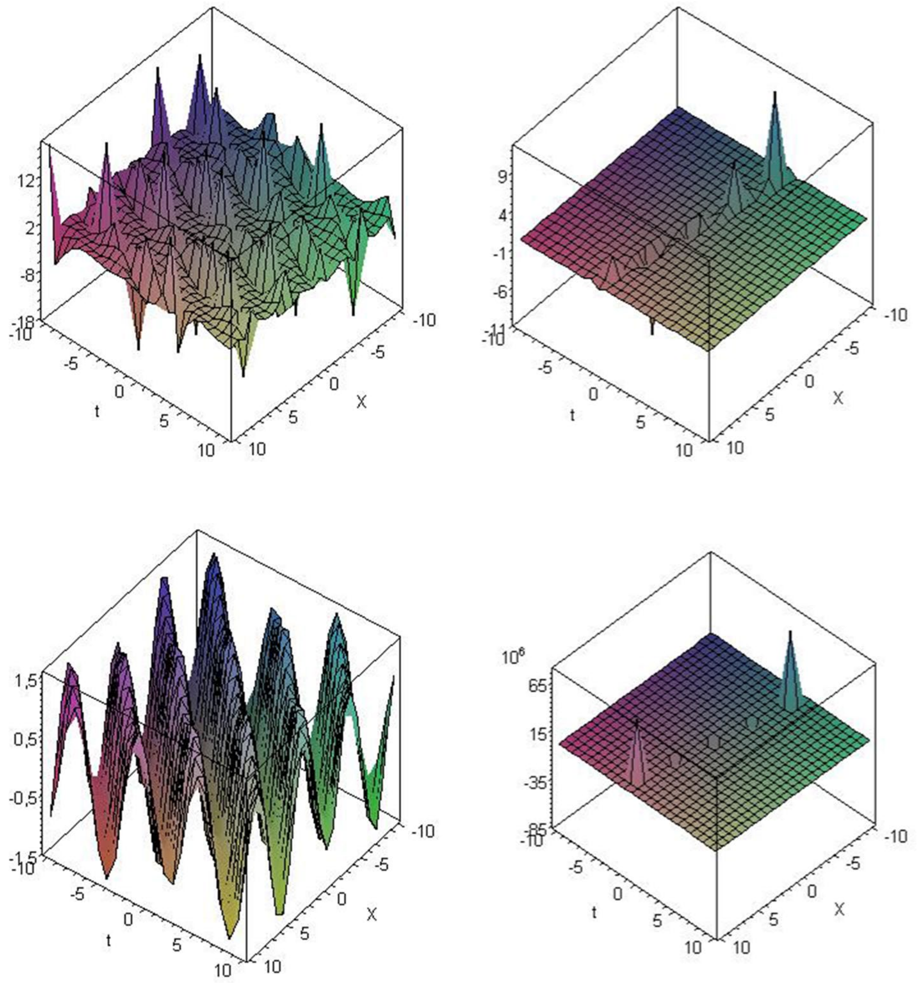

Fig. 1 The 3-D graphs of real part for solutions $u_1(x, y = 0, t)$, $u_3(x, y = 0, t)$, $u_5(x, y = 0, t)$, $u_8(x, y = 0, t)$ are plotted above for the parameter choices $p = 1$, $q = 2$, $r = 3$, $\sigma = 1$, $\kappa = 1$, $v = 1$, $A = 10$, $B = 1$, $C = 1$ for $y = 0$ in $-10 < x < 10$, $-10 < t < 10$

predictable wave character with free parameters used. These graphs are promising in that they explain the exact model of many wave types with the solutions obtained by GUM.

The 3D wave profiles have been displayed to show the temporal and spatial changes in the x direction of the obtained solutions by the GUM as above. As they have many different solutions with varying structure with respect to free parameters, physical structures and graphical illustrations of the some selected solutions for the only some selected parameters of the equation are demonstrated in this section to save space. The varying structures of the solutions in (37) to (44) can be tested by using any Computer Algebra Software for different parameters. However, the wave variable η that supposes propagation of a wave of fixed form is the most fundamental form of analysis that governs the propagation of the waves observed in figures. Additionally, the rate $\frac{F}{D} = \frac{(pq\sigma+v)}{2p\kappa}$ affects the amplitude of the wave due

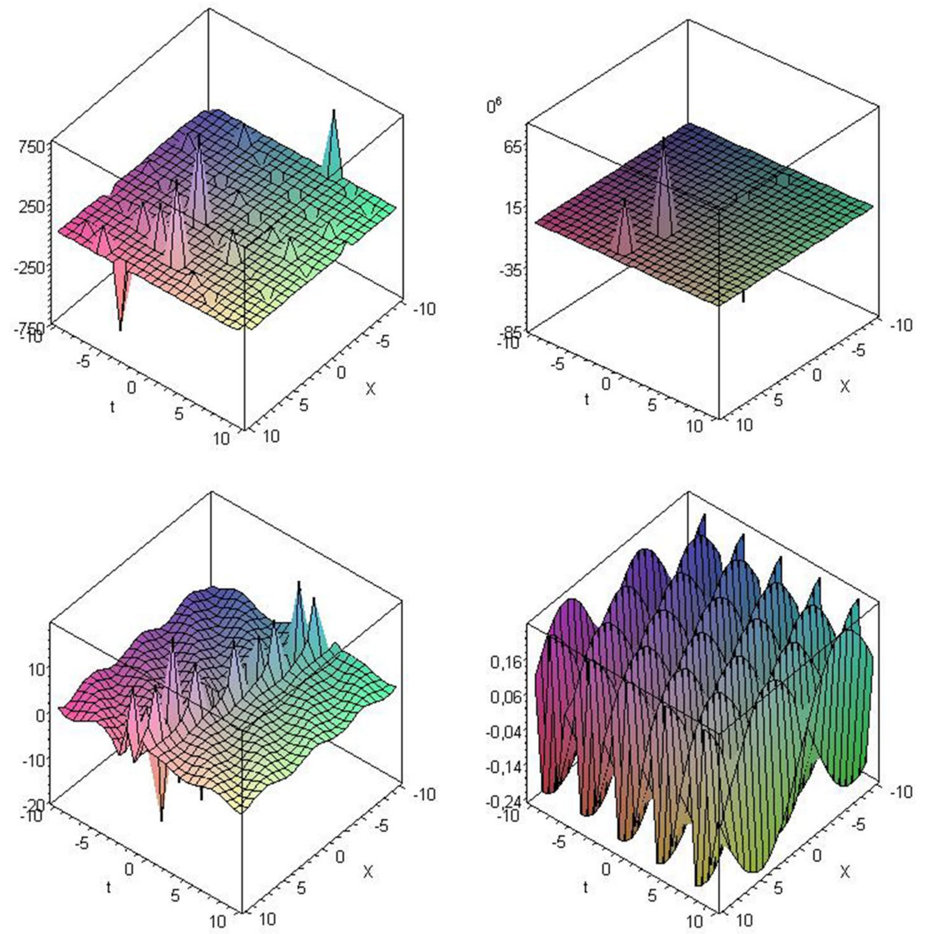

Fig. 2 The 3-D graphs of real part for solution $u_2,\ u_4,\ u_6,\ u_7$ are plotted above for the parameter choices $p = 1,\ q = 2,\ r = 3,\ \sigma = 1,\ \kappa = 1,\ v = 1, A = 10, B = 0,\ C = 0$ for $y = 0$ in $-10 < x < 10,\ -10 < t < 10$

to mathematical structures of the solutions given from u_1 to u_7. Therefore, the amplitude is controlled by this rate with respect to solutions.

5 Conclusions

In this study, the generalized unified method (GUM), a new computational method to construct solutions for the problems in mathematical physics, has been successfully applied to obtain solutions for the Mukherjee-Naskar (KMN) Equation. The main and significant contributions of this article are as follows:

- It is introduced the GUM as a new extension method to solve both NPDEs and fractionel NPDEs.

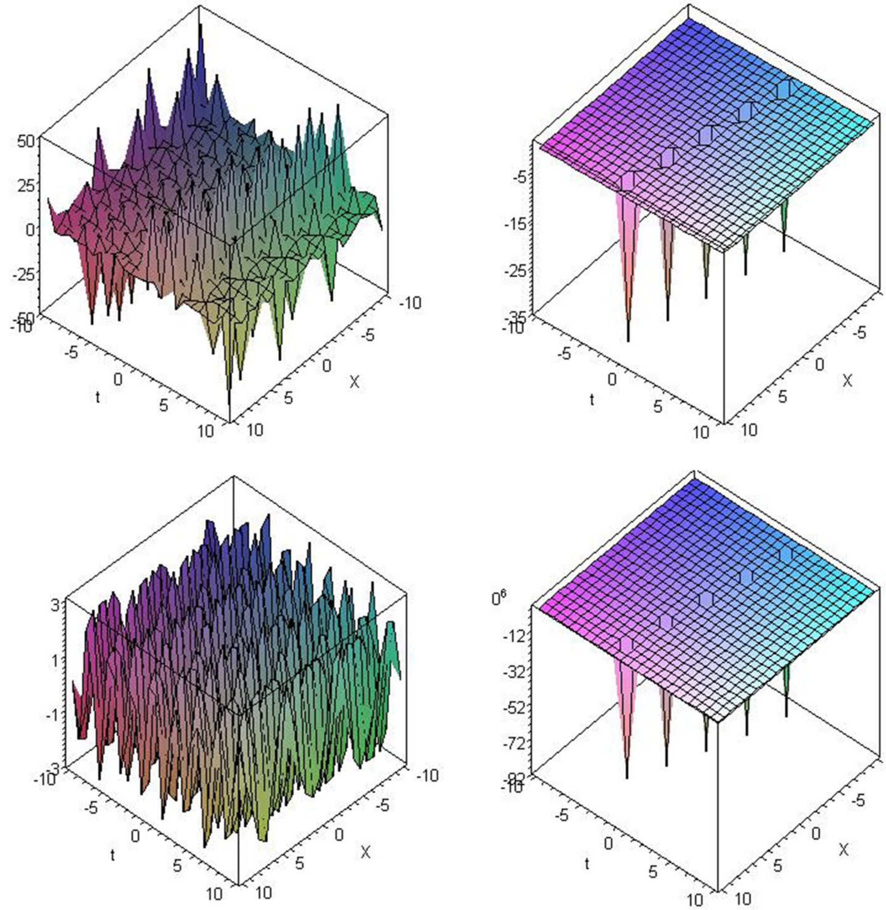

Fig. 3 The 3-D graphs of real part for solution $u_1(x, y = 0, t)$, $u_3(x, y = 0, t)$, $u_5(x, y = 0, t)$, $u_8(x, y = 0, t)$ are plotted above for the parameter choices $p = 1,\ q = 2,\ r = 3,\ \sigma = 1,\ \kappa = 1,\ v = 10$ $, A = 10,\ B = 1,\ C = 0$ for $y = 0$ in $-10 < x < 10,\ -10 < t < 10$

- Applying this method successfully to the KNM equation gives more general solutions in compact form with free parameters $A \neq 0$, B, C and η_0, Ω_0. Therefore, many solutions are retrieved using these free parameters by using only one method without reproducing same solution in different forms.
- The obtained solutions in Sect. 3 are collected under the eight main families with several free parameters in Sect. 4. Moreover, solutions in hyperbolic, trigonometric and rational forms, and many more can be derived from these families by using eleven free parameters which are p, q, r, v,$\sigma \neq 0$,$\kappa \neq 0$, η_0, Ω_0, B, C and $A \neq 0$ as explained in Sect. 4.
- The physical structures of some selected solutions obtained by GUM are plotted for only some selected free parameters by using Maple. Thus, the graphs show that how diverse characteristics structures can be derived from these solution sets.
- Due to free parameters, it can be tuned physical properties of the wave such amplitude, width and velocity. Considering the physical importance of the KMN equation,

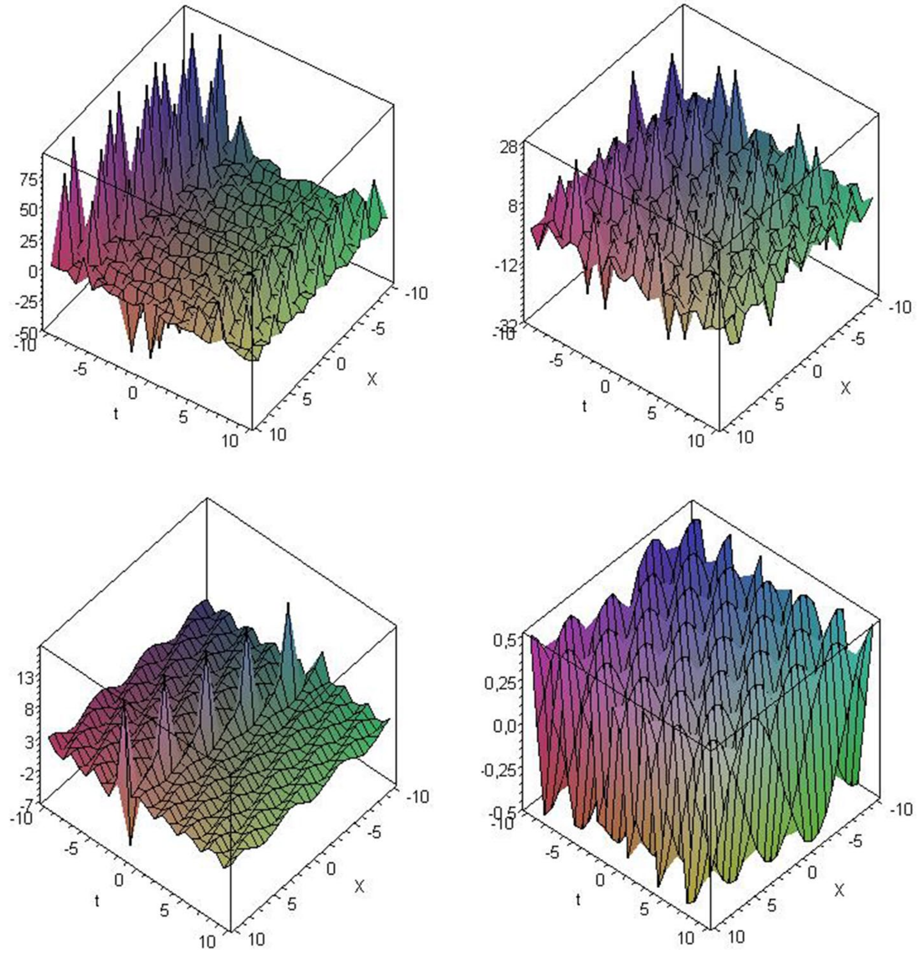

Fig. 4 The 3-D graphs of real part for solution $u_2(x, y = 0, t)$, $u_4(x, y = 0, t)$, $u_6(x, y = 0, t)$, $u_7(x, y = 0, t)$ are plotted above for the parameter choices $p = 1$, $q = 2$, $r = 3$, $\sigma = 1$, $\kappa = 1$, $v = 10$, $A = 10$, $B = 0$, $C = 1$ for $y = 0$ in $-10 < x < 10, -10 < t < 10$

this observation of changes in amplitude and velocity makes a valuable contribution to the applied sciences that use the KMN equation to model their problems.

- Comparing with other methods, the algorithm is very simple and very easy to perform on computer. In other words, more solutions are obtained effortlessly without doing tedious calculations.

The computations in this work have been performed by Maple 12.

Author Contributions All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

Funding There is no funding for this work.

Data Availability Not applicable.

Declarations

Conflict of interest The author declare that there is no competing interests.

Ethical approval The author would like to clarify that there is no financial/non-financial interests that are directly or indirectly related to the work submitted for publication.

References

Akbulut, A., Islam, R., Arafat, Y., Tascan, F.: A novel scheme for SMCH equation with two different approaches. Comput. Methods Differ. Equ. **11**(2), 263–280 (2023)

Akbulut, A., Kumar, D.: Conservation laws and optical solutions of the complex modified korteweg-de vries equation. J. Ocean Eng. Sci. (2022). https://doi.org/10.1016/j.joes.2022.04.022

Akcagil, S., Aydemir, T.: A new application of the unified method. NTMSCI **6**(1), 185–199 (2018)

Akinyemi, L., Senol, M., Akpan, U., Oluwasegun, K.: The optical soliton solutions of generalized coupled nonlinear Schrodinger-Korteweg-de vries equations. Opt. Quanum Electron. **53**, 1–14 (2021)

Ali, I., Seadawy, A.R., Rizvi, S.T.R., Younis, M., Ali, K.: Conserved quantities along with Painleve analysis and optical solitons for the nonlinear dynamics of Heisenberg ferromagnetic spin chains model. Int. J. Mod. Phys. B **34**(30), 2050283 (2020)

Aliyu, A.I., Li, Y., Baleanu, D.: Single and combined optical solitons, and conservation laws in (2+ 1)-dimensions with Kundu-Mukherjee-naskar equation. Chin. J. Phys. **63**, 410–418 (2020)

Althobaiti, A., Althobaiti, S., El-Rashidy, K., Seadawy, A.R.: Exact solutions for the nonlinear extended kdV equation in a stratified shear flow using modified exponential rational method. Results Phys. **29**, 104723 (2021)

Arafat, S.Y., Islam, S.R., Bashar, M.H.: Influence of the free parameters and obtained wave solutions from CBS equation. Int. J. Appl. Comput. Math. **8**(3), 99 (2022)

Arafat, S.Y., Khan, K., Islam, S.R., Rahman, M.: Parametric effects on paraxial nonlinear Schrodinger equation in Kerr media. Chin. J. Phys. (2022). https://doi.org/10.1016/j.cjph.2022.08.026

Asghari, Y., Eslami, M., Rezazadeh, H.: Exact solutions to the conformable time-fractional discretized mKdV lattice system using the fractional transformation method. Opt. Quantum Electron. **55**(4), 318 (2023)

Asghari, Y., Eslami, M., Rezazadeh, H.: Soliton solutions for the time-fractional nonlinear differential difference equation with conformable derivatives in the ferroelectric materials. Opt. Quantum Electron. **55**(4), 289 (2023)

Asjad, M.I., Inc, M., Faridi, W.A., Bakar, M.A., Muhammad, T., Rezazadeh, H.: Optical solitonic structures with singular and non-singular kernel for nonlinear fractional model in quantum mechanics. Opt. Quantum Electron. **55**(3), 1–20 (2023)

Bilal, M., Ahmad, J.: Dynamical nonlinear wave structures of the predator-prey model using conformable derivative and its stability analysis. Pramana **96**(3), 149 (2022)

Bilal, M., Ahmad, J.: Investigation of diverse genres exact soliton solutions to the nonlinear dynamical model via three mathematical methods. J. Ocean Eng. Sci. (2022). https://doi.org/10.1016/j.joes.2022.05.031

Bilal, M., Ren, J., Younas, U.: Stability analysis and optical soliton solutions to the nonlinear Schrodinger model with efficient computational techniques. Opt. Quantum Electron. **53**, 1–19 (2021)

Bilal, M., Rehman, S., Ahmad, J.: Analysis in fiber Bragg gratings with Kerr law nonlinearity for diverse optical soliton solutions by reliable analytical techniques. Mod. Phys. Lett. B **36**(23), 2250122 (2022)

Biswas, A., Ekici, M., Sonmezoglu, A., Zhou, Q., Moshokoa, S.P., Belic, M.: Optical soliton perturbation with full nonlinearity for Kundu–Eckhaus equation by extended trial function schema. Opt.- Int. J. Light Electron Opt. **160**, 17–23 (2018)

Biswas, A., Vega-Guzman, J., Bansal, A., Kara, A.H., Alzahrani, A.K., Zhou, Q., Belic, M.R.: Optical dromions, domain walls and conservation laws with Kundu–Mukherjee–naskar equation via traveling waves and lie symmetry. Results Phys. **16**, 102850 (2020)

Biswas, A., Yildirim, Y., Yasar, E., Triki, H., Zhou, Q., Moshokoa, S.P., Ullah, M.Z., Belic, M., et al.: Optical soliton perturbation with full nonlinearity in polarization preserving fibers using trial equation method. J. Optoelectron. Adv. Mater. **20**(7–8), 385–402 (2018)

Biswas, A., Zhou, Q., Ullah, M.Z., Triki, H., Moshokoa, S.P., Belic, M.: Optical soliton perturbation with anti-cubic nonlinearity by semi-inverse variational principle. Optik **143**, 131–134 (2017)

Bulut, H., Sulaiman, T.A., Demirdag, B.: Dynamics of soliton solutions in the chiral nonlinear Schrodinger equations. Nonlinear Dyn. **91**, 1985–1991 (2018)

Ekici, M., Sonmezoglu, A., Biswas, A., Belic, M.R.: Optical solitons in (2+1)-dimensions with Kundumukherjee- Naskar equation by extended trial function scheme. Chin. J. Phys. **57**, 72–77 (2019)

El-Rashidy, K., Seadawy, A.R.: Kinky breathers, multi-peak and multi-wave soliton solutions for the nonlinear propagation of Kundu–Eckhaus dynamical model. Int. J. Mod. Phys. B **34**(32), 2050317 (2020)

El-Rashidy, K., Seadawy, A.R., Althobaiti, S., Makhlouf, M.: Investigation of interactional phenomena and multi wave solutions of the quantum hydrodynamic Zakharov–Kuznetsov model. Open Phys. **19**(1), 91–99 (2021)

Eslami, M., Neirameh, A.: New exact solutions for higher order nonlinear Schrodinger equation in optical fibers. Opt. Quantum Electron. **50**, 1–8 (2018)

Gozukizil, O.F., Akcagil, S., Aydemir, T.: Unification of all hyperbolic tangent function methods. Open Phys. **14**(1), 524–541 (2016)

Gunerhan, H., Khodadad, F.S., Rezazadeh, H., Khater, M.M.: Exact optical solutions of the (2+ 1) dimensions Kundu–Mukherjee–Naskar model via the new extended direct algebraic method. Mod. Phys. Lett. B **34**(22), 2050225 (2020)

Guo, M., Fu, C., Zhang, Y., Liu, J., Yang, H.: Study of ion-acoustic solitary waves in a magnetized plasma using the three-dimensional time-space fractional schamel-kdv equation. Complexity (2018). https://doi.org/10.1155/2018/6852548

Haas, F., Mahmood, S.: Nonlinear ion-acoustic solitons in a magnetized quantum plasma with arbitrary degeneracy of electrons. Phys. Rev. E **94**(3), 033212 (2016)

Helal, M.A.: Introduction to Solitons, p. 289. Solitons, Giza (2022)

Hollm, M., Dostal, L., Fischer, H., Seifried, R.: Study on the interaction of nonlinear water waves considering random seas. PAMM **20**(1), e202000307 (2021)

Islam, S.R., Bashar, M.H., Arafat, S.Y., Wang, H., Roshid, M.M.: Effect of the free parameters on the Biswas-Arshed model with a unified technique. Chin. J. Phys. **77**, 2501–2519 (2022)

Islam, S.R., Kumar, D., Fendzi-Donfack, E., et al.: Impacts of nonlinearity and wave dispersion parameters on the soliton pulses of the (2+ 1)-dimensional Kundu–Mukherjee–Naskar equation. Rev. Mexicana de Fisica **68**, 061301–06131 (2022)

Jhangeer, A., Seadawy, A.R., Ali, F., Ahmed, A.: New complex waves of perturbed Shrodinger equation with Kerr law nonlinearity and Kundu–Mukherjee–Naskar equation. Results Phys. **16**, 102816 (2020)

Khalil, E.M., Sulaiman, T.A., Yusuf, A., Inc, M.: The m-fractional improved perturbed nonlinear Schrodinger equation: optical solitons and modulation instability analysis. Int. J. Mod. Phys. B **35**(08), 2150121 (2021)

Kudryashov, N.A.: General solution of traveling wave reduction for the Kundu–Mukherjee–Naskar model. Optik **186**, 22–27 (2019)

Kudryashov, N.A.: Optical solitons of the resonant nonlinear Schrodinger equation with arbitrary index. Optik **235**, 166626 (2021)

Kumar, D., Hasan, M.M., Paul, G.C., Debnath, D., Mondal, N., Faruk, O.: Revisiting the spatiotemporal dynamics of a diffusive predator-prey system: an analytical approach. Results in Phys. **44**, 106122 (2023)

Kundu, A.: Novel integrable higher-dimensional nonlinear schroedinger equation: Properties, solutions, applications. arXiv:1305.4023 (2013)

Kundu, A., Mukherjee, A., Naskar, T.: Modelling rogue waves through exact dynamical lump soliton controlled by ocean currents. Proc. Royal Soc. A: Math., Phys. Eng. Sci. **470**(2164), 20130576 (2014)

Lan, Z.-Z., Guo, B.-L.: Nonlinear waves behaviors for a coupled generalized nonlinear Schrodinger-boussinesq system in a homogeneous magnetized plasma. Nonlinear Dyn. **100**, 3771–3784 (2020)

Malik, S., Hashemi, M.S., Kumar, S., Rezazadeh, H., Mahmoud, W., Osman, M.: Application of new Kudryashov method to various nonlinear partial differential equations. Opt. Quantum Electron. **55**(1), 8 (2023)

Marin, M., Seadawy, A., Vlase, S., Chirila, A.: On mixed problem in Thermoelasticity of type iii for Cosserat media. J. Taibah Univ. Sci. **16**(1), 1264–1274 (2022)

Mukherjee, A., Janaki, M., Kundu, A.: A new (2+ 1) dimensional integrable evolution equation for an ion acoustic wave in a magnetized plasma. Phys. Plasmas **22**(7), 072302 (2015)

Mukherjee, A., Kundu, A.: Novel nonlinear wave equation: regulated rogue waves and accelerated soliton solutions. Phys. Lett. A **383**(10), 985–990 (2019)

Nakatsuji, H., Nakashima, H., Kurokawa, Y.I.: Solving the Schrodinger equation of atoms and molecules: chemical-formula theory, free-complement chemical-formula theory, and intermediate variational theory. J. Chem. Phys. **149**(11), 114105 (2018)

Nandi, D.C., Ullah, M.S., Ali, M.Z., et al.: Application of the unified method to solve the ion sound and Langmuir waves model. Heliyon **8**(10), e10924 (2022)

Rezazadeh, H., Kurt, A., Tozar, A., Tasbozan, O., Mirhosseini-Alizamini, S.M.: Wave behaviors of Kundu–Mukherjee–Naskar model arising in optical fiber communication systems with complex structure. Opt. Quantum Electron. **53**, 1–11 (2021)

Saha, A., Pradhan, B., Banerjee, S.: Multistability and dynamical properties of ion-acoustic wave for the nonlinear Schrodinger equation in an electron-ion quantum plasma. Phys. Scr. **95**(5), 055602 (2020)

San, S., Seadawy, A.R., Yasar, E.: Optical soliton solution analysis for the (2+ 1) dimensional Kundu–Mukherjee–Naskar model with local fractional derivatives. Opt. Quantum Electron. **54**(7), 442 (2022)

Seadawy, A.R., Ali, A., Althobaiti, S., El-Rashidy, K.: Construction of abundant novel analytical solutions of the space-time fractional nonlinear generalized equal width model via Riemann–Liouville derivative with application of mathematical methods. Open Phys. **19**(1), 657–668 (2021)

Seadawy, A.R., Cheemaa, N.: Some new families of spiky solitary waves of one-dimensional higherorder k-dV equation with power law nonlinearity in plasma physics. Indian J. Phys. **94**(1), 117–126 (2020)

Shahen, N.H.M., Rahman, M., et al.: Dispersive solitary wave structures with mi analysis to the unidirectional DGH equation via the unified method. Partial Differ. Equ. Appl. Math. **6**, 100444 (2022)

Singh, S., Mukherjee, A., Sakkaravarthi, K., Murugesan, K.: Higher dimensional localized and periodic wave dynamics in an integrable (2+ 1)-dimensional deep water oceanic wave model. Waves Random Complex Med. **33**(1), 78–97 (2023)

Sulaiman, T.A., Akturk, T., Bulut, H., Baskonus, H.M.: Investigation of various soliton solutions to the Heisenberg ferromagnetic spin chain equation. J. Electromagn. Waves Appl. **32**(9), 1093–1105 (2018)

Talarposhti, R., Jalili, P., Rezazadeh, H., Jalili, B., Ganji, D., Adel, W., Bekir, A.: Optical soliton solutions to the (2+ 1)-dimensional Kundu–Mukherjee–Naskar equation. Int. J. Mod. Phys. B **34**(11), 2050102 (2020)

Uddin, S., Karim, S., Alshammari, F., Roshid, H.-O., Noor, N., Hoque, F., Nadeem, M., Akgul, A., et al.: Bifurcation analysis of travelling waves and multi-rogue wave solutions for a nonlinear pseudo-parabolic model of Visco-Elastic Kelvin–Voigt fluid. Math. Probl. Eng. (2022). https://doi.org/10.1155/2022/8227124

Ullah, M.S., Abdeljabbar, A., Roshid, H.-O., Ali, M.Z.: Application of the unified method to solve the Biswas–Arshed model. Results Phys. **42**, 105946 (2022)

Ullah, M.S., Ali, M.Z., Biswas, A., Ekici, M., Khan, S., Moraru, L., Alzahrani, A.K., Belic, M.R., et al.: Optical soliton polarization with Lakshmanan–Porsezian–Daniel model by unified approach. Results Phys. **22**, 103958 (2021)

Wang, J., Shehzad, K., Seadawy, A.R., Arshad, M., Asmat, F.: Dynamic study of multi-peak solitons and other wave solutions of new coupled kdV and new coupled Zakharov–Kuznetsov systems with their stability. J. Taibah Univ. Sci. **17**(1), 2163872 (2023)

Wen, X.: Higher-order rational solutions for the (2+1) dimensional kmn equation. In: Proceedings of the Romanian Academy Series A, 18(3), 191–198. Yildirim, Y. (2019a) (2017)

Yildirim, Y.: Optical solitons to Kundu–Mukherjee–Naskar model in birefringent fibers with trial equation approach. Optik **183**, 1026–1031 (2019)

Yildirim, Y.: Optical solitons to Kundu–Mukherjee–Naskar model with modified simple equation approach. Optik **184**, 247–252 (2019)

Yildirim, Y.: Optical solitons to Kundu–Mukherjee–Naskar model with trial equation approach. Optik **183**, 1061–1065 (2019)

Yildirim, Y., Mirzazadeh, M.: Optical pulses with Kundu–Mukherjee–Naskar model in fiber communication systems. Chin. J. Phys. **64**, 183–193 (2020)